Paper Reference(s) 9MA0/01
Pearson Edexcel Level 3 GCE

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

Tuesday 4 June 2024 – Afternoon

Time: 2 hours

Question Booklet

DO NOT RETURN THIS BOOKLET WITH THE ANSWER BOOKLET.

Y75693A

Pearson

YOU MUST HAVE

Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN

A separate Diagram Booklet
A separate Answer Booklet

INSTRUCTIONS

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet or in the separate Diagram Booklet – there may be more space than you need.

Do NOT write on this Question Booklet.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **15** questions in this question booklet. The total mark for this paper is **100**

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

You may be provided with a model for Question 7. It is NOT accurate.

There may be spare copies of some diagrams.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. $g(x) = 3x^3 - 20x^2 + (k + 17)x + k$

where k is a constant.

Given that $(x - 3)$ is a factor of $g(x)$, find the value of k.

(Total for Question 1 is 3 marks)

2. (a) Find, in ascending powers of x, the first four terms of the binomial expansion of

$$(1 - 9x)^{\frac{1}{2}}$$

giving each term in simplest form.
(3 marks)

(b) Give a reason why $x = -\frac{2}{9}$ should NOT be used in the expansion to find an approximation to $\sqrt{3}$

(1 mark)

(Total for Question 2 is 4 marks)

3. $f(x) = x + \tan\left(\frac{1}{2}x\right) \qquad \pi < x < \frac{3\pi}{2}$

Given that the equation $f(x) = 0$ has a single root α

(a) show that α lies in the interval $[3\cdot6, 3\cdot7]$

(2 marks)

(b) Find $f'(x)$

(2 marks)

(c) Using $3\cdot7$ as a first approximation for α, apply the Newton–Raphson method once to obtain a second approximation for α. Give your answer to 3 decimal places.

(2 marks)

(Total for Question 3 is 6 marks)

4. Given that $y = x^2$, use differentiation from first principles to show that

$$\frac{dy}{dx} = 2x$$

(Total for Question 4 is 3 marks)

5. The function, f, is defined by

$$f(x) = \frac{2x - 3}{x^2 + 4} \qquad x \in \mathbb{R}$$

(a) Show that

$$f'(x) = \frac{ax^2 + bx + c}{\left(x^2 + 4\right)^2}$$

where a, b and c are constants to be found.

(3 marks)

(b) Hence, using algebra, find the values of x for which f is decreasing.

You must show each step in your working.

(3 marks)

(Total for Question 5 is 6 marks)

6. Look at the diagram for Question 6 in the separate Diagram Booklet.
The diagram shows a sketch of the graph with equation

$$y = 3|x - 2| + 5$$

The vertex of the graph is at the point P, shown in the diagram.

(a) Find the coordinates of P.
(2 marks)

(b) Solve the equation given below.

$$16 - 4x = 3|x - 2| + 5$$

(2 marks)

(continued on the next page)

6. continued.

(c) A line l has equation $y = kx + 4$ where k is a constant.

Given that l intersects $y = 3|x - 2| + 5$ at 2 distinct points,

find the range of values of k.

(2 marks)

(Total for Question 6 is 6 marks)

7. Look at the diagram for Question 7 in the separate Diagram Booklet.
The diagram is NOT drawn to scale.
The diagram shows a cylindrical tank of height 1·5 m
A model may be provided for this question.
The model is NOT accurate.

Initially the tank is full of water.

The water starts to leak from a small hole, at a point L, in the side of the tank.

(continued on the next page)

7. continued.

While the tank is leaking, the depth, H metres, of the water in the tank is modelled by the differential equation:

$$\frac{dH}{dt} = -0{\cdot}12e^{-0{\cdot}2t}$$

where t hours is the time after the leak starts.

Using the model,

(a) show that

$$H = Ae^{-0{\cdot}2t} + B$$

where A and B are constants to be found,
(3 marks)

(continued on the next page)

Turn over

7. continued.

(b) find the time taken for the depth of the water to decrease to 1·2 m
Give your answer in hours and minutes, to the nearest minute.
(3 marks)

(c) In the long term, the water level in the tank falls to the same height as the hole.

Find, according to the model, the height of the hole from the bottom of the tank.
(2 marks)

(Total for Question 7 is 8 marks)

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8. The functions f and g are defined by

$$f(x) = 4 - 3x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{5}{2x - 9} \qquad x \in \mathbb{R}, \ x \neq \frac{9}{2}$$

(a) Find fg(2)

(2 marks)

(b) Find g^{-1}

(3 marks)

(c) (i) Find gf(x), giving your answer as a simplified fraction.

(ii) Deduce the range of gf(x).

(3 marks)

(continued on the next page)

8. continued.

(d) The function h is defined by

$$h(x) = 2x^2 - 6x + k \qquad x \in \mathbb{R}$$

where k is a constant.

Find the range of values of k for which the equation

$$f(x) = h(x)$$

has no real solutions.
(3 marks)

(Total for Question 8 is 11 marks)

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9. **The first 3 terms of a geometric sequence are:**

$$3^{4k-5} \qquad 9^{7-2k} \qquad 3^{2(k-1)}$$

where k is a constant.

(a) **Using algebra and making your reasoning clear, prove that $k = \frac{5}{2}$**

(3 marks)

(b) **Hence find the sum to infinity of the geometric sequence.**

(3 marks)

(Total for Question 9 is 6 marks)

10. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Look at the diagram for Question 10 in the separate Diagram Booklet.
The diagram shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \qquad x \geq 0$$

(a) The curve crosses the x-axis at the point A.

Verify that the x coordinate of A is 4
(1 mark)

(continued on the next page)

10. continued.

(b) The line l_1 is the tangent to the curve at A.

Use calculus to show that an equation of line l_1 is:

$$12x + y = 48$$

(3 marks)

(c) The line l_2 has equation $y = 8x$

The region R, shown shaded in the diagram, is bounded by the curve, the line l_1 and the line l_2

Use algebraic integration to find the exact area of R.

(5 marks)

(Total for Question 10 is 9 marks)

11. Look at the diagram for Question 11 in the separate Diagram Booklet.
The diagram shows the design of a badge.

The shape ABCOA is a semicircle with centre O and diameter 10 cm

OB is the arc of a circle with centre A and radius 5 cm

The region R, shown shaded in the diagram, is bounded by the arc OB, the arc BC and the line OC.

Find the exact area of the region R.

Give your answer in the form $(a\sqrt{3} + b\pi)$ cm^2, where a and b are rational numbers.

(Total for Question 11 is 4 marks)

12. (a) Express $140\cos\theta - 480\sin\theta$ in the form $K\cos(\theta + \alpha)$

where $K > 0$ and $0 < \alpha < 90°$

State the value of K and give the value of α, in degrees, to 2 decimal places.
(3 marks)

(continued on the next page)

12. continued.

(b) A scientist studies the number of rabbits and the number of foxes in a wood for one year.

The number of rabbits, R, is modelled by the equation

$$R = A + 140\cos(30t)^\circ - 480\sin(30t)^\circ$$

where t months is the time after the start of the year and A is a constant.

(continued on the next page)

12. (b) continued.

Given that, during the year, the maximum number of rabbits in the wood is 1500

(i) find a complete equation for this model.

(ii) Hence write down the minimum number of rabbits in the wood during the year according to the model.
(2 marks)

(continued on the next page)

12. continued.

(c) The actual number of rabbits in the wood is at its minimum value in the middle of April.

Use this information to comment on the model for the number of rabbits. (2 marks)

(continued on the next page)

12. continued.

(d) The number of foxes, F, in the wood during the same year is modelled by the equation

$$F = 100 + 70\sin(30t + 70)^\circ$$

The number of foxes is at its minimum value after T months.

Find, according to the models, the number of RABBITS in the wood at time T months.
(4 marks)

(Total for Question 12 is 11 marks)

13. (a) Given that a is a positive constant, use the substitution $x = a\sin^2\theta$ to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a - x}\,dx = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}} \sin^2 2\theta\,d\theta$$

(4 marks)

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a - x}\,dx = k\pi a^2$$

where k is a constant to be found.

(4 marks)

(Total for Question 13 is 8 marks)

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14. A balloon is being inflated.

In a simple model,

- the balloon is modelled as a sphere
- the rate of increase of the radius of the balloon is inversely proportional to the square root of the radius of the balloon.

(a) At time t seconds, the radius of the balloon is r cm

Write down a differential equation to model this situation.
(1 mark)

(continued on the next page)

14. continued.

(b) At the instant when $t = 10$

- **the radius is 16 cm**
- **the radius is increasing at a rate of $0\cdot9\ \text{cm s}^{-1}$**

Solve the differential equation to show that

$$r^{\frac{3}{2}} = 5\cdot4t + 10$$

(5 marks)

(continued on the next page)

14. continued.

(c) Hence find the radius of the balloon when $t = 20$

Give your answer to the nearest millimetre.
(2 marks)

(d) Suggest a limitation of the model.
(1 mark)

(Total for Question 14 is 9 marks)

15. (i) Show that $k^2 - 4k + 5$ is positive for all real values of k.
(2 marks)

(ii) A student was asked to prove by contradiction that

"There are no positive integers x and y such that

$$(3x + 2y)(2x - 5y) = 28\text{"}$$

(continued on the next page)

15. (ii) continued.

The start of the student's proof is shown below.

Assume that positive integers x and y exist such that:

$$(3x + 2y)(2x - 5y) = 28$$

If $3x + 2y = 14$ and $2x - 5y = 2$

$$\left.\begin{aligned} 3x + 2y &= 14 \\ 2x - 5y &= 2 \end{aligned}\right\} \Rightarrow$$

$$x = \frac{74}{19},\ y = \frac{22}{19} \text{ Not integers}$$

(continued on the next page)

15. (ii) continued.

Show the calculations and statements needed to complete the proof.
(4 marks)

(Total for Question 15 is 6 marks)

END OF PAPER
TOTAL FOR PAPER IS 100 MARKS